P^2, dec 10th 2003

Some considerations regarding transverse-to-longitudinal phase space exchange at FNPL

At AAC meeting last Nov 19th, during my presentation, A. Zholents brought to my attention we have all the elements (in the final version of the upgrade) to test the "transverse to longitudinal emittance exchange" scheme proposed by Cornacchia and Emma (C&E) in [PRSTAB(5):084001 (2002)].

The basic idea is to construct a transfer matrix that has has non zero element in its *z*-*y* 2X2 sub-matrix. If the transport matrix for the (y,y',z,δ) coordinate writes:

$$R_{4\times 4} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Then with the use of simplecticity of R along with properties of the determinant one can show that the vertical and longitudinal emittance are mapped via R as:

$$\epsilon_y^2 \xrightarrow{R} |A|^2 \epsilon_y + (1-|A|^2)\epsilon_z^2 + 4\sigma_{y'}^2 \sigma_\delta^2$$
$$\epsilon_z^2 \xrightarrow{R} (1-|A|^2)\epsilon_y + |A|^2 \epsilon_z^2 + 4\sigma_{y'}^2 \sigma_\delta^2$$

where $|A|$ stands for the determinant of the matrix A.

Thus if $|A|=0$ vertical and longitudinal emittances can be exchanged.

Transverse-to-longitudinal phase space exchange

In C&E it is proposed to build the *R* matrix with non zero *B* and *C* sub-matrices by locating a deflecting cavity in a dispersive section.

- A dispersive section introduces coupling between y,y' and δ,
- A deflecting cavity introduces coupling between y' and z and between δ and y.

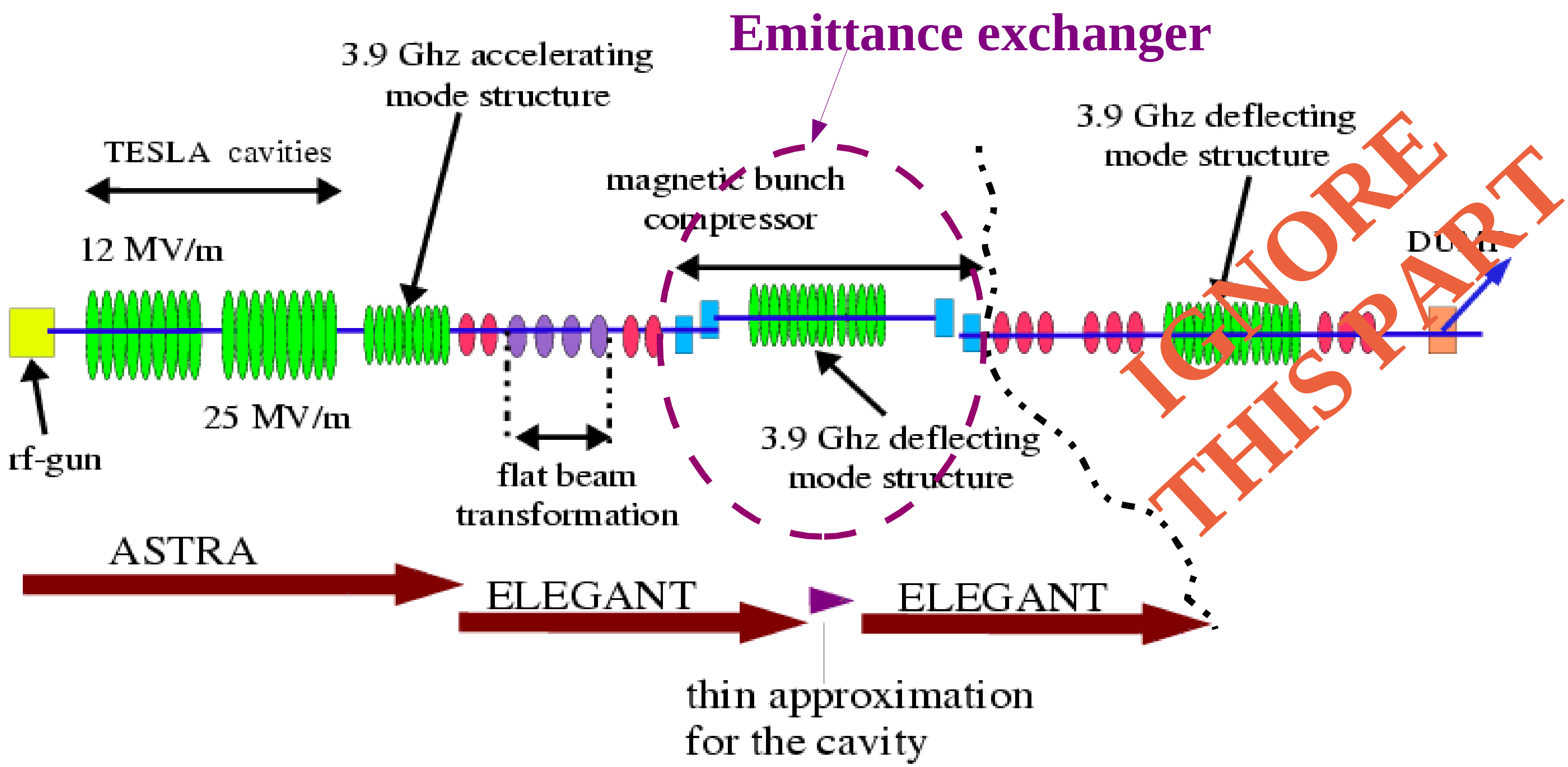
It is indeed proposed to locate the deflecting cavity at the mid-point of a magnetic chicane similar to those use for bunch compression. From the derived transfer matrix for the total system is is also shown that in order to have a total emittance exchange $|A|=0$ in the previous slide implies the condition:

$$\eta_y k = 1$$

where k characterizes the deflecting cavity (see next slides) and η_y is the vertical dispersion at the chicane mid-point (FNPL chicane bends vertically).

At FNPL we have both a magnetic chicane and possibly a deflecting cavities!

Simple model: Astra + Elegant + thin lens approximation model



- Astra used from cathode to exit of 3.9 Ghz monopole mode cavity
- Elegant used elsewhere apart for the deflecting cavity located in the chicane
- Simulation stops downstream of the chicane

Thin lens approximation for the deflecting cavity

(the deflecting model cavity in elegant does not incorporate *Ez* variation on *y* so one has to have his own model)

A macro-particle with coordinate $(x_i, x_i', y_i, y_i', t_i, \gamma_i)$ will experience, while passing through the vertically deflecting cavity, an energy and vertical angular kick given by:

Energy kick: $$\Delta\gamma_i = \frac{eV}{mc^2\langle E\rangle a}\langle\gamma\rangle y_i \equiv k\langle\gamma\rangle y_i$$

Transverse angular kick: $$\Delta y_i' = \frac{eVc}{\langle E\rangle a} t_i \equiv kc\,t_i$$

These coordinates transformations are performed on the Elegant output in the chicane mid-point and re-injected has an input into Elegant.

In the above equations, *V* stands for the longitudinal accelerating voltage in the deflecting cavity at an offset *a*. <*E*> is the average incoming beam energy. The far right identities are to define *k* accordingly to C&E.

From L. Bellantoni: $V/a = 2040$ MV.m^{-1} (for a [integrated] deflecting voltage of 2.5 MV).

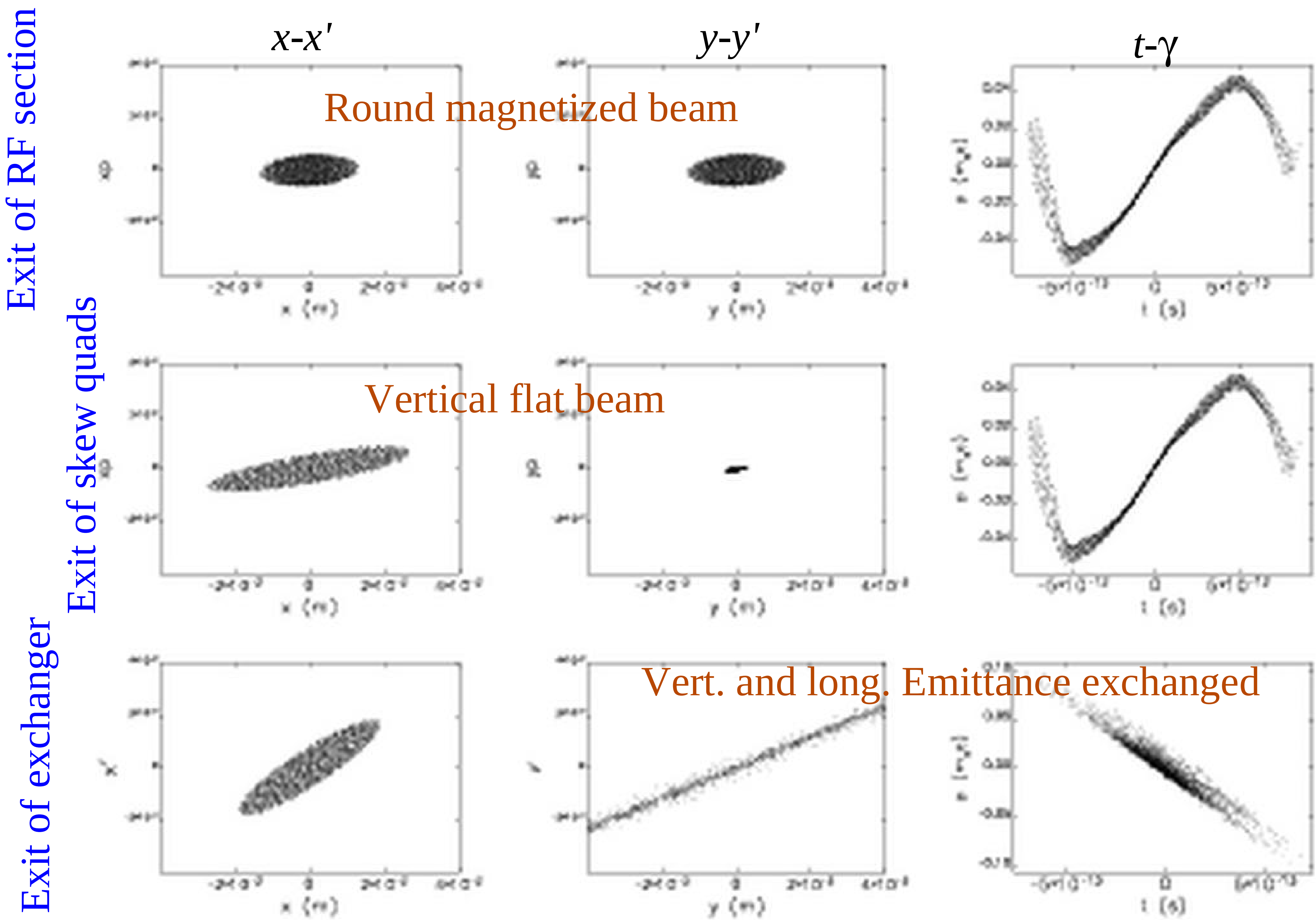
Emittance exchange

In order to satisfy the condition $k\eta_y = 1$ that is $V/a = \eta_y \langle E \rangle / e$ given the dispersion, η_y=0.38 m, and the maximum beam energy of approximately 50 MeV, we would need a maximum value of $V/a \approx 20$ MV.m^{-1} which would correspond to a very small value of the integrated transverse deflecting voltage of about 0.025 MV (this maybe achievable with a smaller (one cell?) deflecting cavity?)

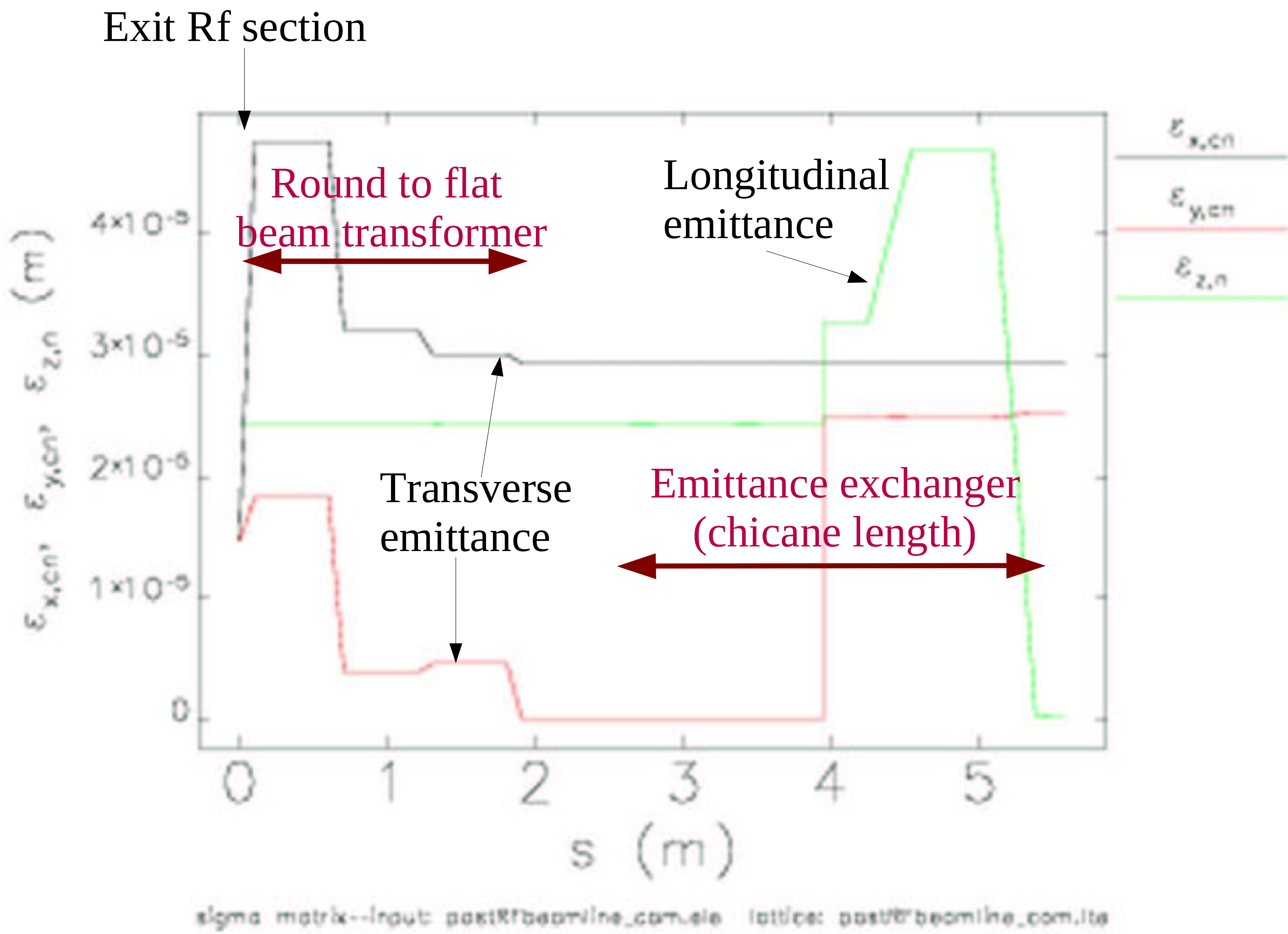
Consider a nominal set-up for flat beam, the emittance downstream of the flat beam transform are:
ε_x=29.4 µm, ε_y=0.12 µm, and ε_z=24.4 µm the operating condition are laser pulse length of 3 ps on cathode, longitudinal phase linearized and phase of rf-systems tuned to minimized total energy spread

So contrary to (not very conservative) assumption of C&E, we do not have a longitudinal emittance much smaller that the transverse emittance. However we could swap the vertical (flat) plane emittance with the longi-tudinal one to at least demonstrate the scheme experimentally.

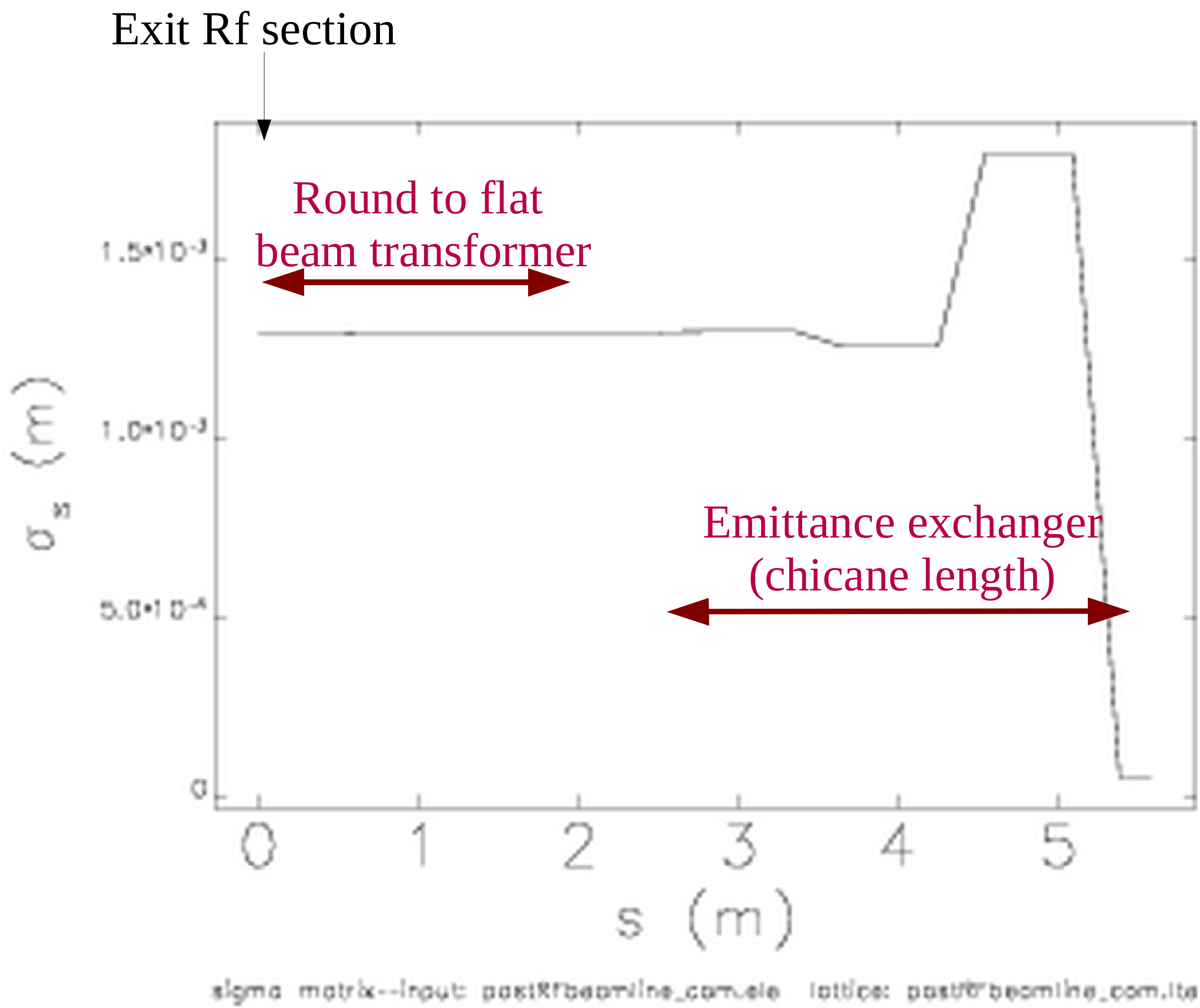
Phase spaces evolution single-particle tracking



Emittances evolution - single-particle tracking



Bunch length - single-particle tracking



Conclusions & further work

- At FNPL, we can exchange the longitudinal to one of the transverse emittance,
- The best to observe a clear signature would be to exchange the smallest emittance of a flat beam with its longitudinal emittance. This would results in a strong reduction of longitudinal emittance and strong bunching that we should be able to characterize with the Martin-Puplett interferometer,
- Of course the best would be to reduce the largest of the flat beam transverse emittance, but the longitudinal emittance (in the set-up I used) was too large to gain from such an exchange,
- The is room for optimization (i.e. Going to lower charge, longer laser pulse, might help)
- Next step is to refine the calculation including space charge everywhere by tracking using `synergia` a fast 3D space charge code based on `impact3D` and `mxyzptl` toolbox (I am currently interacting with P. Spenzouris regarding this topic)

- ***IDEA:*** **maybe we could slightly elongate the compressor I proposed in the context of the upgrade in order to include a small deflecting cavity (perhaps just one of the copper model would be sufficient for a proof-of-principle experiment).**